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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Mechanics 2 (6678/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

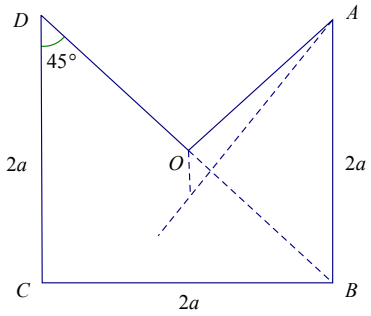
  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

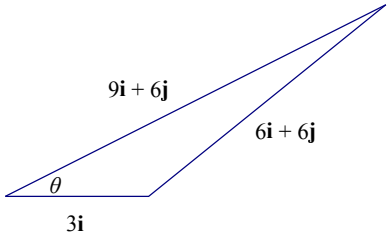
  RHS, LHS Right hand side, left hand side.

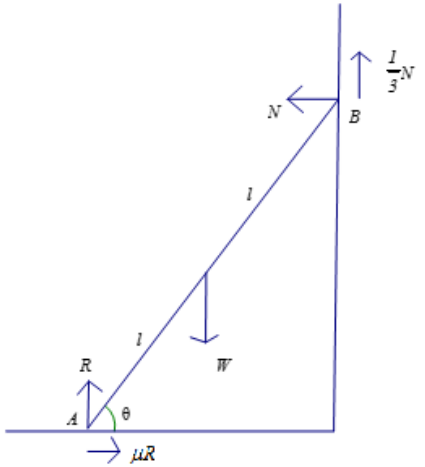
June 2015
6678 Mechanics 2
Mark Scheme

Question Number	Scheme	Marks	
1.	$12500 = 5F$	B1	Use of $P = Fv$
	$F + 900g\sin\theta - 570 = 900a$	M1	Use of $F = ma$ parallel to the slope. All 4 terms required. Condone sign errors and sin/cos confusion. 12500 in place of F is M0 – dimensionally incorrect
		A2	Correct unsimplified equation. -1 each error
	$a = 2.47 \quad (2.5)$	A1	
			Working with the positive direction up the slope is acceptable for the first 4 marks, but their final answer must be positive.
		[5]	

Question Number	Scheme	Marks	
2a	Ratio of masses 1 : 3 : 4	B1	Correct ratios for their division Also common: 3 equal triangles, 6 equal triangles, a rectangle and two equal triangles
	Centre of mass of triangle $\frac{2}{3}a$ from O	B1	Correct centres for the triangles in their division consistent with their axis.
	Moments about horizontal axis through O: $4\times 0-1\times\frac{2}{3}a=3d$	M1	Condone 4×0 not seen. Terms must be of correct form Condone use of moments about a parallel axis Signs must be consistent with their axis and distances. Watch out for people who add a triangle to the square.
	$\left(d=-\frac{2}{9}a\right)$ Distance = $\frac{2}{9}a$	A1	Reach ***Given answer**** with no errors seen. Their answer must be positive
		[4]	
2b			
	$\sqrt{2}akM=\frac{2}{9}a\cos 45M$	M1	Moments about A Lengths must be resolved as necessary (need use of trig).
		A2	-1 each error
	$k=\frac{1}{9}$	A1	
		[4]	

Alt 2b	Take A as origin and axes along AD and AB, use moments to find components of distance of c of m from A.	M1	Could choose a different origin
	$\bar{x} = \dfrac{a(1+2k)}{1+k}$	A1	
	$\bar{y} = \dfrac{11a}{9(1+k)}$	A1	
	$\bar{x} = \bar{y} \Rightarrow \dfrac{11}{9} = 1+2k \Rightarrow k = \dfrac{1}{9}$	A1	CWO. This mark is not available if they have assumed that the c of m of the system is at O.
Alt 2b	$\dfrac{2}{9}a\cos 45 = \dfrac{1}{9}.\sqrt{2}a = \dfrac{1}{9}OD$	M1A1	Using ratios
	$kM \times OD = M \times \dfrac{1}{9}OD$	A1	
	$\Rightarrow k = \dfrac{1}{9}$	A1	

Question Number	Scheme	Marks	
3a	: $0.75\mathbf{v} = 6\mathbf{i} + 6\mathbf{j} + 0.75\times 4\mathbf{i}\ \ (= 9\mathbf{i} + 6\mathbf{j})$	M1	Impulse momentum equation Must be considering $\pm m(\mathbf{v} - \mathbf{u})$
		A1	Correct unsimplified
	$\mathbf{v} = 12\mathbf{i} + 8\mathbf{j}$	A1	Award in (a) if seen or if (a) is completed correctly. Award in (b) if (a) is incomplete and this mark has not been awarded and correct **v** seen for the first time in (b)
			Could have a velocity triangle rather than momentum, in which case the vectors are $4\mathbf{i}, 8\mathbf{i} + 8\mathbf{j}, 12\mathbf{i} + 8\mathbf{j}$
	$\theta = \tan^{-1}\left(\frac{2}{3}\right)$ or $\theta = \cos^{-1}\left(\frac{1+13-8}{2\sqrt{13}}\right)$, or equivalent	M1	Correct trig to find the required angle
	$33.7°$ or 0.588 radians	A1	Accept $34°$ or better . Must be the final answer
		[5]	
3a alt	$\begin{pmatrix}6\\6\end{pmatrix} = 0.75\begin{pmatrix}v\cos\theta\\v\sin\theta\end{pmatrix} - 0.75\begin{pmatrix}4\\0\end{pmatrix}$	M1A1	
	$\Rightarrow 0.75\times v\cos\theta = 9,\ \ 0.75\times v\sin\theta = 6$	A1	
	$\Rightarrow \tan\theta = \frac{2}{3}$	M1	
	$33.7°$ or 0.588 radians	A1	
3b	Change in KE $= \frac{1}{2}\times\frac{3}{4}(144+64) - \frac{1}{2}\times\frac{3}{4}(16)$	M1	Finding a difference between KE terms. Must **use** $\frac{1}{2}mv^2$ in both terms (all of v, not just one component of it.)
		A1ft	follow their **v.** Allow ±
	=72 (J)	A1	CAO
		[3]	
		(8)	

Question Number	Scheme	Marks	
4			NB: If μ and $\frac{1}{3}$ are used the wrong way round the candidate loses the first A1 and the final A1.
	Resolve horizontally or vertically:	M1	Allow without friction $= \mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction . Condone Wg
	Take moments about A or B.	M1	All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A): 2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B): \ 2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error. Could be in terms of Fs. -1 if see Wg in place of W. Any Friction force used should be acting in the right direction. Mark the equation, not what they have called it.
	$\frac{10}{3}N + \frac{2}{3}N = W$ or $2R = W + 2\mu R \times \frac{5}{3}$	M1	Use $\tan\theta = 5/3$ (substitute values for the trig ratios)
	$\Rightarrow 4N = W \Rightarrow 4N - R = \frac{1}{3}N$	DM1	Equation in N and R (Eliminate one unknown) Dependent on the moments equation
	$\frac{11}{3}\mu R = R$	DM1	Solve for μ Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better

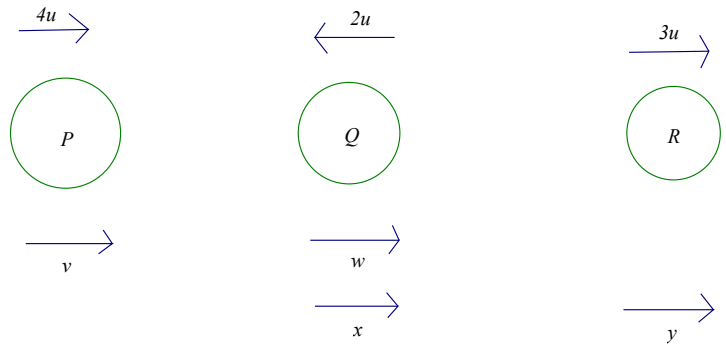
Alt 1	Resolve horizontally or vertically:	M1	Allow without friction $= \mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction
		M1	Take moments about A or B. All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A): 2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B): \ 2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error, Could be in terms of Fs. -1 if Wg used. Mark the equation, not what they have called it. Any Friction force used should be acting in the right direction. For this method they need two moments equations – allows the marks for their best equation.
	$2lN\sin\theta + 2l\frac{N}{3}\cos\theta = 2l\cos\theta R - \mu R 2l\sin\theta$	DM1	Use two moments equations to eliminate W Dependent on the moments equation
	Use of $\tan\theta$: $2\mu\times\frac{5}{3} + \frac{2}{3}\mu = 2 - 2\mu\times\frac{5}{3}$	M1	Substitute for the trig ratios
	Solve for μ: $\left(\frac{20}{3} + \frac{2}{3}\right)\mu = 2,$	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better

Alt 2	Resolving horizontally or vertically:	M1	Allow without friction $= \mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction (condone Wg)
	$l\cos\theta \times R = l\cos\theta \times \frac{1}{3}N + l\sin\theta \times N + l\sin\theta \times \mu R$	M1	Moments about the centre of the rod. All terms required. Terms must be resolved. Condone sign errors and sin/cos confusion. Allow without friction $= \frac{1}{3}N$. Any Friction force used should be acting in the right direction.
		A2	-1 each error. Could be in terms of Fs. -1 if Wg used.
	$l\cos\theta \times R = l\cos\theta \times \frac{1}{3}\mu R + l\sin\theta \times \mu R + l\sin\theta \times \mu R$	DM1	Obtain an equation in μ and θ $\left(\cos\theta = \cos\theta \times \frac{1}{3}\mu + \sin\theta \times \mu + \sin\theta \times \mu\right)$ Dependent on the moments equation
	$\cos\theta\left(1 - \frac{1}{3}\mu\right) = 2\mu\sin\theta \Rightarrow \quad \tan\theta = \frac{1 - \frac{1}{3}\mu}{2\mu} = \frac{5}{3}$	M1	Use of $\tan\theta$ (substitute values for the trig ratios)
	Solve for μ : $10\mu = 3 - \mu$,	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better
		[9]	

Question Number	Scheme	Marks	
5a	Max friction $= \mu \times 10g\cos\alpha$	B1	Use of μR seen or implied $(\mu \times 90.46)$
	Work done against friction $= 6.5 \times 10g\mu\cos\alpha\,(=245)$	M1	For 6.5 x their F.
	Equation in μ: $6.5 \times 10g\mu \times \frac{12}{13} = 245$,	DM1	Dependent on preceding M1.
		A1	Correct substituted equation
	$\mu = 0.417$ or 0.42	A1	Do not accept $\frac{5}{12}$ (cannot have an exact value following the use of 9.8) Use of 9.81 is a rubric infringement, so A0 if seen.
		[5]	
5b	$\frac{1}{2} \times 10 \times 11.5^2 - 245 - 10g \times 6.5\sin\alpha = \frac{1}{2} \times 10 \times v^2$ or equivalent	M1	**Must be using work-energy equation**. All terms required. Condone sign errors and sin/cos confusion.
		A2	-1 each error $(661.25 - 245 - 245 = 171.25)$
	$v = 5.85$ (5.9) (m s^{-1})	A1	
		[4]	
		(9)	

Question Number	Scheme	Marks	
6a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=\left(2t-1\right)\left(t-4\right)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2},\ 4$	A1	Incorrect answers with no method shown score M0A0
		[3]	
6b	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=4t-9$	M1	Differentiate v to obtain a (at least one power of t going down)
		A1	Correct derivative
	$t=5,\ a=11\,(\mathrm{m\,s^{-2}})$	A1	
		[3]	
6c	$s=\int v\mathrm{d}t=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\,(+C)$	M1	Integrate v to obtain s (at least one power of t going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for t in (0,5) from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_0^{\frac{1}{2}}-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_{\frac{1}{2}}^4$ $+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t\left(+15\right)\right]_4^5$	A1	NB: $\int_0^5 v\mathrm{d}t$ scores M0A0A0
	$(0,\ \frac{23}{24},\ -\frac{40}{3},\ \frac{-55}{6})$ $=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=19.4$ (m) $\left(15,\ 15\frac{23}{24}\left(\frac{383}{24}\right),\ \frac{5}{3},\ 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}$ $\left(\frac{233}{12}\right)$ or better
		(11)	

Question Number	Scheme	Marks	
7a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	Vertical component of speed at $A = u + at$	M1	Complete method using *suvat* to find *v*.
	$= u\sin\theta - 4g$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$u\cos\theta = 15\cos 20$ $u\sin\theta = 15\sin 20 + 4g$	DM1	Form simultaneous equations in u and θ and attempt to solve for u or θ. Depends on the previous M1
	$\theta = 72.4 \quad (72)$	A1	Remember - A0 for the first overspecified answer
	$u = 46.5 \quad (47)$	A1	
		[7]	
Alt7a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	At $t = 4$, $s = vt - \frac{1}{2}gt^2$	M1	Complete method to find the vertical height at A
	$= 98.9.....$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2gh$	DM1	Conservation of energy. The equation needs to include all three terms but condone sign error(s).
	$u = 46.5 \quad (47)$	A1	Remember - A0 for the first overspecified answer
	$\theta = 72.4 \quad (72)$	A1	Beware inappropriate use of *suvat*
7b	$-15\sin 20 = 15\sin 20 - gt$ or $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1	Complete method using *suvat* or otherwise to find the time to travel from A to B
	$t = 1.05$ (s) or 1.0 (s)	A1	
		[2]	
7c	Total time $= 4 + (1.05) + 4$	B1ft	Follow their t or $\frac{2u\sin\theta}{g}$ for their u, θ
	Range $= 46.5\times\cos 72.4\times(8+1.05)$ (or $15\cos 20\times 9.05$)	M1	Correct method to find OC for their t,u and θ
	$= 128$ (m) or 127 (m) (130)	A1	
		[3]	
		(12)	

Question Number	Scheme	Marks	
8a			
	$4mu - 2mu = mv + mw$	M1	Equation for CLM. Requires all 4 terms. Condone sign errors. Condone m missing throughout.
		A1	
	$w - v = 6eu$	M1	Impact law. e must be used correctly. Condone sign errors
		A1	Signs should be consistent with equation for CLM.
	$v + w = 2u$ $w - v = 6eu \qquad 2w = 2u + 6eu$, $(w = u + 3eu)$	DM1	Solve for w. Dependent on the two previous M marks.
	For Q and R to collide require $w > 3u$,	M1	Use **inequality** to compare their w with $3u$.
	$u + 3eu > 3u$, $\qquad e > \frac{2}{3}$	A1	Reach ***Given answer*** with no errors seen.
		[7]	

8a alt	Collision between Q and $R \Rightarrow w > 3u$	M1	
	Magnitude of impulse on $Q > 5mu$		
	Magnitude of impulse on $P > 5mu$	A1	
	$\Rightarrow v < -u$	M1	
	$e = \dfrac{w-v}{4u+2u}$	M1	Impact law
	Speed of separation after collision $> u + 3u$	M1	
	$e > \dfrac{4u}{4u+2u}$	A1	
	$e > \dfrac{2}{3}$	A1	Reach given inequality with no errors seen.
8b	$w = u + 3eu = \dfrac{13}{4}u$, $v = -\dfrac{5}{4}u$	B1	Correct values seen or implied.
	$mw + 3mu = mx + my$ $\dfrac{3}{4}(w-3u) = y - x$	M1	Equations for CLM and impact. Allow with w or their w. All terms required. Condone sign errors. e must be used correctly.
		A1	Correct equations in w or their w.
	$\dfrac{25}{4}u = x + y$, $\dfrac{3u}{16} = y - x$		
	Solve for x (or kx)	DM1	Dependent on the previous M1 Need to get far enough to give a convincing concluding argument.
	$\dfrac{97}{16}u = 2x$, $x = \dfrac{97u}{32}\left(= 3.03125u\right)$	A1	Correct expression for kx
	P and Q moving away from each other, so no collision.	A1	Reach **given conclusion** with no errors seen.
		[6]	
		(13)	

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